The model based on the diffusion equation is very valuable. It gives good results, although the values are somewhat less than the measured ones, when the surface temperature of the deposit is below the melting point, which was so in our experiments.

NOTATION

K, k_m, effective thermal conductivity of cryodeposit and cold wall, respectively; ρ_f, ρ_s, density of cryodeposit and monolithic solid phase, respectively; r_c, sublimation heat; h, heat-transfer coefficient; h_m, mass-transfer coefficient; δ, thickness of cryodeposit; D, diffusion coefficient; ρ_{va}, ρ_{vs}, vapor density in volume and at surface temperature; T_a, T_w, T_s, T_m, temperature in volume, of the cold wall, of surface, and melting, respectively; P, P_v, P_F, total pressure, partial vapor pressure in volume, and saturation pressure at the surface temperature; k_s, thermal conductivity of the monolithic solid phase; τ, crookedness, $\tau = 1.1$; ε, porosity; B_1, B_2, B_3, parameters in formula (5).

LITERATURE CITED

1. G. Biguria and L. A. Wenzel, "Measurement and correlation of water frost thermal conductivity and density," I.E.C. Fundamentals, 9, No. 1, 129-138 (1970).
2. M. A. Dietenberger, "Generalized correlation of the water frost thermal conductivity," Int. J. Heat Mass Transfer, 26, No. 4, 607-619 (1983).
3. H. W. Schneider, "Frost growth rate equation, forming on cold surfaces," Int. J. Heat Mass Transfer, 21, No. 8, 1019-1024 (1978).
4. R. F. Barron and L. S. Han, "Heat and mass transfer to a cryogenic surface under free-convection conditions," Teploenergetika, No. 4, 84-92 (1965).

ICING ON THE WALLS OF A BURIED PIPELINE BEARING A FREEZING NON-NEWTONIAN LIQUID

V. M. Ovsyannikov

UDC 536.421

The definitive parameters have been derived. Approximate formulas have been obtained for the maximum ice thickness, the time to attain the maximum, and the melting time.

In recent years, a technology has been developed in this country and elsewhere for transporting finely divided highly concentrated suspensions of coal in water [1, 2]. The mixtures travel in pipes in laminar mode or in some cases turbulent. They may have non-Newtonian or pseudoplastic rheological features.

In the design of hydrotransport systems for the harsh climate of Siberia, there is the novel problem of hydrotransport for freezing liquids. The startup period in the winter is particularly hazardous from the viewpoint of freezing, when the soil around the pipeline has not yet been heated and has a temperature below the freezing point of the liquid.

The fullest analysis of the icing problem in the Russian literature is to be found in [3]. In [4], icing calculations were compared with measurements for laminar flow of a Newtonian liquid, and a confirmation of the model formulation was obtained. In [3, 4], there are mainly calculations on the steady-state icing. Ice growth and thawing have been examined in [5, 6]. We now present the formulation.

The pipeline is buried in ground whose temperature at the start is below the freezing point of the liquid. A layer of frozen suspension is formed on the inside, which for brevity

---

All-Union Research and Development Institute for the Pipeline Hydrotransport Industry.
Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 50, No. 2, pp. 285-293, February, 1986. Original article submitted November 27, 1984.

 0022-0841/86/5002-0212$12.50

we call the ice layer. As the surrounding soil heats up, the leakage of heat into it decreases, and the layer of ice may melt completely. A water—coal mixture has viscoelastic properties and obeys the Svedov—Bingham law or the law of pseudoplastic liquid [7]. The rheological features may be described by the Ostwald de Wil power-law model with parameter k < 1, while in some cases k may differ only slightly from 1:

$$\tau' = \mu \left| \frac{\partial u}{\partial r} \right|^{k-1} \left(\frac{\partial u}{\partial r} \right).$$

The temperature varies little across the cross section of the pipe, so μ and k can be taken as constant along the radius. The solution to the equation of motion for laminar flow is obtained for a tube with the following velocity profile:

$$\frac{u}{u_{m0}} = \frac{3k+1}{k+1} \left[1 - \left(\frac{r}{R} \right)^{(k+1)/k} \right], \tag{1}$$

where u_{m0} is the mean speed in a part of the tube free from ice. In a part where there is an ice crust of thickness δ, the velocity profile is

$$\frac{u}{u_m} = \frac{3k+1}{k+1} \left[1 - \left(\frac{r}{R-\delta} \right)^{(k+1)/k} \right]. \tag{2}$$

To calculate the flow and heat transfer under turbulent conditions, we use the turbulent model of [8]. The temperature pattern in the liquid is given by the energy equation

$$c \frac{\partial T}{\partial t} + cu \frac{\partial T}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r} \left(r\lambda \frac{\partial T}{\partial r} \right) + \tau' \frac{\partial u}{\partial r}, \tag{3}$$

where the thermal conductivity λ and coefficient of friction τ' are determined on the basis of the flow being laminar or turbulent.

When there is icing in the laminar state, this equation on the basis of (2) becomes

$$c \frac{\partial T}{\partial t} + cu_{m0} \frac{R^2}{(R-\delta)^2} \frac{3k+1}{k+1} \left[1 - \left(\frac{r}{R-\delta} \right)^{(k+1)/k} \right] \frac{\partial T}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r} \left(r\lambda \frac{\partial T}{\partial r} \right) + \mu \left(\frac{3k+1}{k} \right)^{k+1} \left(\frac{r}{R-\delta} \right)^{(k+1)/k} \frac{u_{m0}^{k+1} R^{2+2k}}{(R-\delta)^{3+3k}}. \tag{4}$$

The initial and boundary conditions are

$$T(r) = T_b \text{ for } x = 0,\ 0 \leqslant r \leqslant R,$$

$$\frac{\partial T}{\partial r} = 0 \text{ for } r = 0$$

at the wall without ice (δ = 0)

$$-\lambda \frac{\partial T}{\partial r} = (T_w - T_\infty) \lambda_s Q/R \text{ at } r = R, \tag{5}$$

and with ice

$$T_w = T_c,\ H \frac{\partial \delta}{\partial t} = -\lambda \frac{\partial T}{\partial r} + (T_c - T_\infty) \lambda_s Q/(R-\delta). \tag{6}$$

To calculate the heat flux entering the ice and the surrounding soil, we solve the thermal-conduction equation, which gives the temperature pattern for a cylindrical source of constant surface temperature T_w embedded in an unbounded medium with initial temperature T_∞. The solution given in [9] for large and small values of the dimensionless time τ was supplemented with a formula to interpolate to the intermediate range $0.5 \leq \tau \leq 3.16$. The heat-transfer coefficient α for the cylindrical source of radius ρ in a medium with thermal conductivity λ_s is given by

$$\alpha(t) = \lambda_s Q(\tau)/\rho,$$

where

$$Q = \begin{cases} (\pi\tau)^{-0.5} + 0.5 - 0.25(\tau/\pi)^{0.5} + \tau/8 \text{ for } \tau \leqslant 0.5, \\ 1.0835\tau^{-0.2186} \text{ for } 0.5 < \tau < 3.16, \\ \dfrac{2}{\ln(4\tau) - 2\gamma} - \dfrac{2\gamma}{[\ln(4\tau) - 2\gamma]^2}, \ \gamma = 0.57722 \text{ for } \tau \geqslant 3.16, \end{cases} \tag{7}$$

$$\tau + \tau_0 = a_s \int_0^t [R - \delta(t)]^{-2} dt, \ \tau_0 = a_s x/(R^2 u_m)$$

Here t is the time reckoned from the moment when the liquid is admitted to the initial section of the pipe, while τ_0 is the dimensionless time corresponding to the liquid attaining section x. We can use this solution for the temperature pattern in the solid phase (in the ice and surrounding ground) subject to the following assumptions:

1) the radius of the cylindrical source varies only slightly in time by comparison with the characteristic heating radius in the solid phase;

2) T_w in the part without ice varies little in time by comparison with $|T_c - T_\infty|$;

3) the thermal conductivity λ_s and thermal diffusivity a_s are the same for the ice and the surrounding ground.

The first two assumptions can be checked when the problem has been solved, and in general are obeyed. The later condition in general of course is not obeyed for various combinations of soils and liquids. However, it substantially simplifies the solution and enables one to obtain the simple and clear analytic expression given below in dimensionless form, which is dependent in the main on three dimensionless parameters. The analysis of this simplified solution is extremely important in elucidating the behavior of the general solution.

Condition 3) is obeyed for certain combinations of liquids and soils. For example, for water ice λ_s = 2 W/(m·deg), while $a_s = 1.4 \times 10^{-6}$ m²/sec, while the values for soils are in the the ranges 1.7-3 and 0.8×10^{-6}-1.6×10^{-6} correspondingly in units with the same dimensions. To derive an analytic solution in more general form without assumption 3), one can introduce two further dimensionless parameters: the ratios of the thermal conductivities and thermal diffusivities for the ice and soil, the solution being obtained by the same method. Therefore, (4)-(7) define the temperature pattern in the liquid in laminar flow and the thickness of the ice. Information on the thermophysical properties of water—coal mixtures is given in [10].

Heat transfer in a solid is slower than that in a moving liquid, and one expects that the nonstationary aspect of the temperature pattern in the liquid will arise mainly from the nonstationary boundary condition at the wall. Therefore, to solve the energy equation, we discarded the term $c\partial T/\partial t$ as small by comparison with the others. Numerical calculations, which are discussed below, show that the $c\partial T/\partial t$ term is less by about a factor of 100 than the convective term $cu\partial T/\partial x$, if we take the mean temperature over the flow as T.

We introduce the dimensionless variable

$$X = \frac{x}{RPe}, \ \mathrm{Pe} = \frac{2Ru_{m0}}{a}, \quad s = \frac{r}{R - \delta}, \quad \theta = \frac{T - T_c}{T_b - T_c}, \ z = \frac{\delta}{R}.$$

Then (4) takes the fairly complicated form if δ is taken as variable:

$$\frac{3k+1}{2(k+1)}\left(1 - s^{(k+1)/k}\right)\left\{\frac{s}{1-z}\frac{\partial z}{\partial X}\frac{\partial \theta}{\partial s} + \frac{\partial \theta}{\partial X} - \frac{2a_s}{a}\frac{\partial \theta}{\partial \tau} + \right.$$

$$\left. + \left[\frac{a_s}{R^2}\int_0^t \frac{2}{(1-z)^3}\frac{\partial z}{\partial X}dt\right]\frac{\partial \theta}{\partial \tau}\right\} = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial \theta}{\partial s}\right) + \left(\frac{3k+1}{k}\right)^{k+1}\frac{\mu u_{m0}^{k+1}}{\lambda(T_b - T_c)R^{k-1}}\frac{s^{(k+1)/k}}{(1-z)^{1+3k}}.$$

To obtain simple solutions suitable for practical use, we examine a simpler equation obtained by discarding terms containing derivatives of the temperature with respect to time and of the ice thickness with respect to the X coordinate.

The simplified equation should give solutions sufficiently close to the true one for small ice thicknesses. These solutions enable one to estimate the discarded terms and indicate when such simplifications cannot be made.

The problem of (4)-(7) for the laminar case takes the form

$$\frac{3k+1}{2(k+1)}(1-s^{(k+1)/k})\frac{\partial\theta}{\partial X}=\frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial\theta}{\partial s}\right)+\left(\frac{3k+1}{k}\right)^{k+1}\frac{Ms^{(k+1)/k}}{(1-z)^{1+3k}}, \tag{8}$$

$$\theta(s)=1 \text{ at } X=0,\ 0\leqslant s\leqslant 1,$$

$$\partial\theta/\partial s=0 \text{ for } s=0,$$

$$\frac{\partial\theta}{\partial s}=-\left(\frac{\theta}{L}+\frac{1}{B}\right)Q(\tau),\ z=0 \text{ for } s=1,\ X\leqslant X_0,$$

$$\tau_0=\frac{2X}{A},\ \theta=0,\ z(X,\ \tau_0+\tau)=BD\int_0^\tau\left(\frac{\partial\theta}{\partial s}\right)\frac{d\tau}{1-z}+D\int_0^\tau\frac{Q(\tau)\,d\tau}{1-z} \text{ for } s=1,\ X>X_0. \tag{9}$$

The system contains the following five dimensionless parameters:

$$D=\frac{C_s(T_c-T_\infty)}{H},\ B=L\frac{T_b-T_c}{T_c-T_\infty},$$

$$M=\mu\frac{u_{m0}^{k+1}}{\lambda(T_b-T_c)R^{k-1}},\quad L=\frac{\lambda}{\lambda_s},\quad A=\frac{a}{a_s}.$$

Here X_0 is the X coordinate where the icing begins and M is a parameter characterizing the heat produced by friction.

We first obtain an approximate analytic solution. If we are given the heat flux incident from the liquid at the ice q_w, we can find z from (9). We use the dimensionless variables to get

$$\frac{q_w R}{\lambda_s(T_c-T_\infty)}=-\theta'_{sw}B.$$

We integrate (9) for small icing thickness $z \ll 1$ and for q_w constant in time to get the τ dependence of z:

$$z/D=\Phi(\tau)+\theta'_{sw}B\tau,\quad \Phi(\tau)=\int_0^\tau Q(\tau)\,d\tau. \tag{10}$$

We derive $dz/d\tau$ and equate it to zero to get the dimensionless time τ_* for the maximum icing and the maximum thickness δ_* of the ice:

$$\tau_*=Q_{-1}(-\theta'_{sw}B),\ \delta_*/(RD)=\Phi(\tau_*)-\tau_*Q(\tau_*). \tag{11}$$

Here Q_{-1} denotes the function inverse to $Q(\tau)$. We write (10) with δ_* and τ_* in the form

$$\frac{\delta(\tau)}{\delta_*}=\frac{\Phi(\tau)-\tau Q(\tau_*)}{\Phi(\tau_*)-\tau_*Q(\tau_*)}.$$

With $\delta = 0$, we get the dimensionless time for the end of icing τ_e as

$$\tau_e=\Phi(\tau_e)/Q(\tau_*),$$

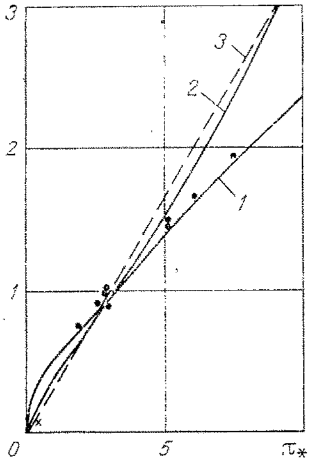

Fig. 1

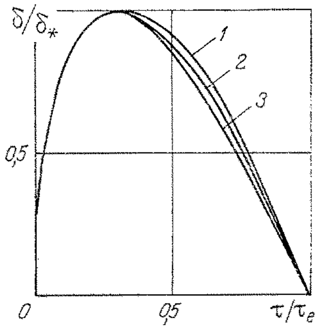

Fig. 2

Fig. 1. Dependence of the end of icing τ_e and the maximum thickness δ_*/R on the time to the attainment of maximum icing in dimensionless form: 1) $\delta_*/(RD)$; 2) $\tau_e/10$; 3) $\tau_e = 3.33\ \tau_*$.

Fig. 2. Dependence of the ice thickness δ/δ_* on τ/τ_e for various values of τ_* in dimensionless form: 1) $\tau_* = 2.5$; 2) $\tau_* = 5$; 3) $\tau_* = 10$.

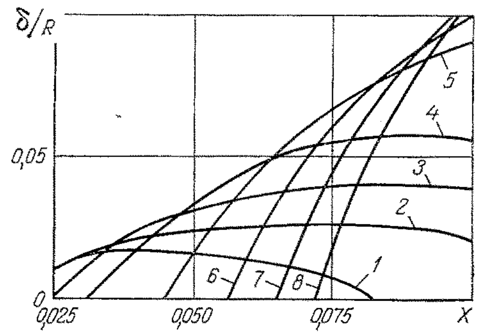

Fig. 3

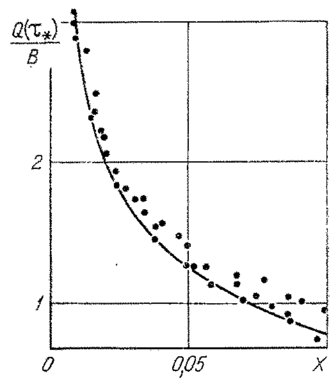

Fig. 4

Fig. 3. Dependence of the relative ice thickness δ/R on coordinate X for various values of the time $\tau + \tau_0$: 1) $\tau + \tau_0 = 2$; 2) 4; 3) 5; 4) 10; 5) 20; 6) 30; 7) 40; 8) 50.

Fig. 4. Dependence of the dimensionless heat flux at the time of maximum icing on the coordinate X along the pipe.

which can be solved by successive approximation. The iteration converges because $|\partial[\Phi(\tau_e)/Q(\tau_*)]/\partial\tau_e| < 1$ if the initial value of $\tau_e > \tau_*$. About 15 iterations are required to calculate τ_e with an absolute accuracy of 0.001.

The solid lines in Fig. 1 show the dependence of τ_e and δ_* on τ_*; the symbols correspond to δ_* obtained in numerical calculations with B varying over the range 0.23-0.8. The two relationships are monotonically increasing, so the thickness of the ice increases the later the maximum occurs, and the icing correspondingly terminates later.

The function $\tau_e(\tau_*)$ can be approximated with an absolute error of 0.1 by the linear relationship $\tau_e = 3.33\tau_*$ shown in Fig. 1 by the dashed line. This shows that the time for which the ice thickens is less by about a factor 2 than the thawing time. The thickness normalized to the maximum is shown as a function of τ normalized to τ_e in Fig. 2, which is a universal function only slightly dependent on τ_*.

We now consider the numerical solution. The problem of (8) and (9) contains the nonlinear equation (8), since the function z appearing in it is dependent on $\partial\theta/\partial s$. However, for $M \ll 1$ or for $z \ll 1$, (8) is close to linear, so the iterative method is effective. Then (8) was solved by the pivot method using a different scheme [11] of approximation accuracy $O(\Delta s^4) + O(\Delta X^2)$, along with the calculation of the integrals with respect to time in (9). The calculations were performed with $\Delta X = 0.003$ along the tube $\Delta\tau = 0.02$ in time, and step along the radius 0.1.

We calculated the temperature distribution at the start of the tube from (8) with a boundary condition of the third kind in the absence of ice on the wall. The calculation gave the dimensionless temperature θ of the liquid at the wall and the derivative of this with respect to s. If the liquid temperature at the wall becomes less than the freezing point, (8) for the next section in X was solved with the boundary condition corresponding to icing. On passing to the next sections along X, it is possible for the icing to cease because of the heating produced by the dissipation. If the ice thickness becomes negative, (8) is again solved with the boundary condition of the third kind corresponding to the absence of ice.

Figure 3 shows the variation in the relative ice thickness δ/R along the pipe for various instants for a Newtonian liquid (k = 1) with D = 0.015; B = 0.466; M = 0.004; L = 0.2; A = 0.08; the time $\tau + \tau_0$ was reckoned from the moment when the liquid entered the initial section. The ice thickness at each instant increases quite smoothly along X and then falls to zero in a section that the liquid has not yet attained. As X increases, q_w monotonically decreases. Therefore, in accordance with (11), the maximum of the icing in time increases as X increases. The Nusselt number Nu and the dimensionless mean flow temperature θ_m along X are

$$\mathrm{Nu} = \frac{2Rq_w}{\lambda (T_m - T_w)}, \quad T_m = 2\int_0^1 sTuds, \; \theta_m = \frac{T_m - T_c}{T_b - T_c}$$

and behave quite smoothly at the icing front. In all cases, they agree quite well with the solution to the Nusselt–Gretz problem with a boundary condition of the first kind for the corresponding value of M.

We now discuss the possibility of using the temperature pattern for a cylindrical source in a solid of (7). This solution does not incorporate precisely the history of the temperature pattern around the pipe. In particular, in the part freed from ice, the solution does not recall the fact that a temperature $T = T_c$ was maintained at the surface of the cylindrical source during the icing. Therefore, the numerical solution shows a temperature step at the wall at the X coordinate corresponding to the icing front. In the form of calculation shown in Fig. 3, this step constitutes 10% of $T_b - T_c$, which is not very large, and which indicates that the solution derived from (7) can be used in practice.

We now discuss the magnitudes of the terms discarded from the energy equation containing $\partial z/\partial X$. Figure 3 shows that $\partial z/\partial X \sim 1$ for the periods when the ice arises and vanishes. Therefore, refined calculations are required for the start and end of the icing. No such refined calculations are required for the start and end of the icing. No such refinement is required for the part in which the ice varies little in thickness along the section having $\partial z/\partial X \sim 0.1$.

Numerical calculations were applied to 14 models with the dimensionless parameters in the following ranges: B = 0.1-0.8; D = 0.0075-0.015; L = 0.1-0.4; A = 0.08; M = 0.001-0.02. In all cases, M was less than 0.02, which characterizes the heat production by friction. With out realistic values for B and D for a major pipeline, a very thick ice crust arises with laminar flow, which makes states with large M technically unfeasible.

The calculations were processed in accordance with the approximate solution discussed above and are shown in Fig. 4. For certain X, we used the ice pattern $\delta(X, \tau)$ to determine τ_* and then to calculate $Q(\tau_*)$. In accordance with the first formula in (11), $Q(\tau_*)/B$ is equal to the derivative of the dimensionless temperature θ'_{sw}, which is a function of X and M, and it is expressed for a laminar flow of a Newtonian liquid via the solution to the Nusselt–Gretz problem with boundary conditions of the first kind [12].

Figure 4 gives results obtained for various values of the dimensionless parameters, which all lie closely around a single curve. The symbols denote points for different D between 0.0075 and 0.03. We conclude that the time to maximum icing is independent of D. The solid line in Fig. 4 is the solution to the Nusselt–Gretz problem with M = 0, since M took small values in the numerical calculations.

The symbols in Fig. 1 indicate δ_* referred to RD as a function of τ_*. These points lie closely around the $\delta_*/(RD)$ curve representing the approximate solution to (11). Therefore, δ_* is dependent only on D and τ_*.

Therefore, only B, M, and D have substantial effects on the icing pattern out of the five parameters D, B, L, M, and A. Here B and M determine the dimensionless time of maximum icing, while D determines the ice thickness. The other parameters have little effect on the solution in the ranges used, where A/2 determines the rate of tube filling with liquid in the dimensionless coordinates (X, τ), while L is an independent parameter only in the part free from ice and thus has an indirect effect on the icing.

The above analysis enables one to determine the thickness at time $\tau + \tau_0$ at the point with coordinate X. For the given X and the calculated B and M, one derives $Q(\tau_*)$ from $\theta'_{sw}(X, M) = Q(\tau_*)$, and then from $\tau = Q_{-1}(Q)$ one determines the corresponding τ_*. From the calculated D and Fig. 1, one determines δ_* at X and the corresponding τ_e. Then Fig. 2 is used to determine δ.

Calculations were made on the dimensionless heat flux $\theta'_{sw}(X)$ for a power-law model with k = 0.11 and k = 0.25. In a certain initial part of the tube, θ'_{sw} due to a non-Newtonian liquid with k < 1 exceeds that for a Newtonian one. These values for the heat flux as applied to the initial part of the tube (X < 0.125) gave more prolonged icing and thicker ice for the Newtonian liquid than for the non-Newtonian one with k < 1. The converse applies in the rest of the tube X > 0.125.

Similar conclusions can be drawn on the icing for tubulent flow. Numerical calculations were performed from (3) with the model of [8], which showed that the dissipative effect in turbulent flow produces much more change in the temperature pattern near the wall than in the laminar mode.

The icing can be calculated by deriving $\theta'_{sw}(X, M)$ for the turbulent state and using it with Fig. 1 and 2 to find τ_*, δ_*, τ_e, and $\delta(\tau)$.

This formulation does not incorporate the effects on the temperature pattern in the soil from the surface at a distance h from the axis of the pipe. As the heat-transfer coefficient from the pipe to the surroundings takes a steady-state value after a certain time, which is defined by the Vlasov—Forheimer formula [13]:

$$\alpha = \lambda_s/[R\ln(2h/R)],$$

we conclude that the results are applicable if τ_* is less than the time required for the heat-transfer coefficient α to attain its steady-state value. Therefore, the solutions are applicable if the burial depth is not very small and satisfies the condition

$$h \geqslant \frac{R}{2}\exp\left[\frac{2}{BNu(X)\theta_m(X)}\right].$$

NOTATION

T, temperature; T_c, fluid freezing temperature; θ, dimensionless temperature; a, thermal diffusivity; c, specific heat; c_s, specific heat of ice; t, time; τ, dimensionless time; τ', friction; δ, ice layer thickness; δ_*, time of maximum ice thickness; R, tube radius; x, coordinate along pipeline; X, dimensionless coordinate along the axis; u, current fluid velocity; u_m, mean velocity; H, specific melting heat of ice; h, depth of axis. Subscripts: w, wall; m, mean; s, ground or ice; b, initial state.

LITERATURE CITED

1. E. P. Olofinskii, "Hydromechanics of pipeline systems for the hydrotransport of solid materials," Izv. Akad. Nauk SSSR, Energ. Transport, No. 4, 29-36 (1979).
2. Burning Very Wet Fuel in the Form of Water-Coal Suspensions [in Russian], Nauka, Moscow (1967).
3. P. A. Bogoslovskii, The Icing Conditions in Pipelines for Hydroelectric Stations [in Russian], Gosénergoizdat, Moscow—Leningrad (1950).
4. R. D. Zerkle and I. E. Sutherland, "The effect of liquid solidification in a tube upon laminar-flow heat transfer and pressure drop," Trans. ASME, Ser. C, Heat Transfer, 90, No. 2, 183-190 (1968).
5. B. A. Krasovitskii, "Thermal and hydraulic conditions in a pipeline transporting a freezing liquid," Inzh.-Fiz. Zh., 35, No. 1, 125-132 (1978).

6. V. M. Zhidkikh and Yu. A. Popov, Icing Conditions in Pipelines [in Russian], Énergiya, Leningrad (1979).
7. V. V. Trainis, Parameters and Modes in the Hydraulic Transportation of Coal in Pipelines [in Russian], Nauka, Moscow (1970).
8. N. I. Buleev and R. Ya. Mironovich, "Heat transfer in a turbulent flow of liquid in a triangular rod lattice," Teplofiz. Vys. Temp., 10, No. 5, 1031-1038 (1972).
9. H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids, Clarendon Press, Oxford (1959).
10. A. A. Agroskin, Coal Physics [in Russian], Nedra, Moscow (1965).
11. I. V. Petukhov, "Numerical calculations on two-dimensional flows in boundary layers," in: Numerical Methods of Solving Differential and Integral Equations with Quadrature Formulas [in Russian], Nauka, Moscow (1964), pp. 304-325.
12. B. S. Petukhov, Heat Transfer and Resistance in the Laminar Flow of a Liquid in a Pipe [in Russian], Énergiya, Moscow (1967).
13. E. P. Shubin, "Heat losses from a pipe buried in the ground," Izv. Vsesoyuznogo Teplotekh. Inst., No. 4, 43-56 (1934).

THE INITIAL STAGE IN CAPILLARY IMPREGNATION

A. V. Kuz'min, P. A. Novikov, and V. I. Novikova

UDC 532.68

The impregnation kinetics are considered during the initial stage of capillary rise. A formula is derived describing the nonstationary impregnation with allowance for inertia, friction, and change in wetting angle.

Porous materials are widely used in space technology, particularly in devices that transport energy and mass (thermal tubes, porous evaporative heat exchangers, etc.), which gives considerable interest to liquid flow in capillaries and porous bodies, while the results are of practical significance.

The static theory of capillary impregnation is reflected in a large number of theoretical and experimental papers, but there has been inadequate research on impregnation kinetics in the initial stages. We therefore analyze the initial stage in the movement of a liquid through a capillary, which plays an important part in starting up and shutting down heat-engineering devices.

The theory of capillary impregnation is based on equation describing the nonstationary laminar flow in a cylindrical capillary [1]:

$$\frac{d^2l}{dt^2}+\frac{1}{l}\left(\frac{dl}{dt}\right)^2+\frac{8\eta}{r^2\rho}\frac{dl}{dt}-\frac{2\sigma\cos\theta}{r\rho l}+g\sin\alpha=0. \quad (1)$$

In solving (1), one often [1-5] neglects the first two terms, which incorporate the inertial force, which in practical cases are less by an order of magnitude than the other terms, i.e., one assumes that

$$\frac{1}{l}\left(\frac{dl}{dt}\right)^2\approx 0,\quad \frac{d^2l}{dt^2}\approx 0 \quad (2)$$

and thus reduces the solution of (1) to the following relation for a horizontal capillary:

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 50, No. 2, pp. 294-299, February, 1986. Original article submitted November 23, 1984.

